Strong-field point-particle limit and the equations of motion in the binary pulsar

著者	二間瀬　敏史
journal or publication title	Physical review. D
volume	36
number	2
page range	321-329
year	1987
URL	http://hdl.handle.net/10097/35589

doi: 10.1103/PhysRevD.36.321

PHYSICAL REVIEW D
PARTICLES AND FIELDS

THIRD SERIES, VOLUME 36, NUMBER 2 15 JULY 1987

Strong-field point-particle limit and the equations of motion in the binary pulsar

T. Futamase
McDonnell Center for the Space Sciences and Department of Physics, Washington University, St. Louis, Missouri 63130
(Received 27 January 1987)

We construct an asymptotic approximation in general relativity to a binary system of two neutron stars by taking a point-particle limit along a sequence of spacetimes. A detailed calculation of the near-zone field is performed in radiation-reaction order. We calculate the rate of change of Newtonian energy and the angular momentum. The result coincides with the standard formula. The equation of spin precession is also calculated and the same result as for weak gravity is obtained. We thus extend the strong equivalence principle for bodies with strong internal gravity to the generation of gravitational radiation and the spin precession.

I. INTRODUCTION

The discovery of the binary pulsar[1] PSR1913 + 16 has led to new activity in the study of equations of motion for a self-gravitating system within general relativity. In particular, many discussions have been devoted to the validity of the quadrupole formula for radiation reaction[2] and many approximation schemes have been developed to derive the formula within the framework of general relativity.[3] We have proposed a new asymptotic approximation scheme by studying a C^∞ sequence of solutions to Einstein's equations that are defined by initial data.[4] By choosing initial data having a "Newtonian" scaling property and defining the limiting process appropriately, we were able to show that the Newtonian and post-Newtonian approximations are genuine asymptotic approximations to general relativity.[5] It is, however, obvious that one cannot apply the post-Newtonian approximation naively to the binary pulsar system because of the stars' strong internal gravity (both stars are believed to be neutron stars). In general relativity one believes the strong equivalence principle to hold which says that the equations of motion for a self-gravitating body are independent of its internal structure and composition up to tidal interactions. Experimental evidence for this was obtained by the lunar Eötvös experiment.[4] Although this evidence applies to bodies with weak internal gravity, it is natural to expect that the principle applies also in the case of strong internal gravity.[7] In the case of the binary pulsar system this means that strong internal gravity in the component stars is irrelevant to the orbital motion which is governed by weak interbody fields, and thus the quadrupole formula derived in weak gravity should still be applicable if one replaces the Newtonian mass of each star by the Schwarzschild mass.

This expectation was supported by recent work of several workers.[3] None of these treatments was, however, able to indicate that the results are asymptotic to any sequence of regular solutions of Einstein's equations. We will show this by explicitly constructing a sequence of spacetimes containing regions of strong gravity and confirm that the above expectation is indeed the case.

The traditional way to incorporate strong internal gravity into general relativity is to use a δ-function-type source with a fixed mass in Einstein's equations. However, this makes Einstein's equations mathematically meaningless, and it is not clear at all how to set up the initial-value problem. Physically, there is no such source in general relativity because of the existence of black holes. Before a body shrinks to a point, it forms a black hole whose size is fixed by its mass. For this reason it has been said that the point particle does not exist in general relativity. This conclusion is not correct, however. All we need is to keep the compactness (M/R), i.e., the strength of the internal field fixed as the body shrinks. This means that we should scale the mass M to go to zero in proportion to the radius R. In such a scaling, the characteristic size R of the body tends to zero, but a black hole never forms because the mass also tends to zero in such a way that $2M/R<1$. This can be fitted into the Newtonian scaling of Ref. 4 very conveniently because there the mass also scales along the sequence of solutions. If we scale the orbital velocity as ϵ and adopt ϵ as a sequence parameter, then the mass must scale as ϵ^2 (fixing the binary separation). Therefore we must scale the size of each body as ϵ^2 in order to keep the compactness of each body fixed. This implies ϵ^{-4} scaling for the densities instead of the ϵ^2 scaling in the Newtonian theory.

The above argument suggests that appropriate initial data to define the sequence would be two uniformly rotating fluids with compact support whose stress-energy tensor scales like ϵ^{-4} and whose size scales like ϵ^2. In the case of rapidly rotating stars the velocity of the fluid has two parts, one having no dependence on ϵ, representing the spinning motion, and the other scaling like ϵ, representing the orbital motion. For slowly rotating stars both parts of the velocity scale like ϵ. We choose the data so that each of these fluid configurations would be a stationary equilibrium solution of Einstein's equation if the other were absent. This choice of data is required to suppress irrelevant internal motions of each body. Any remaining motions are the tidal effects caused by the other body, which will be of order ϵ^6 smaller than the internal self-force. Technically this allows one to use the Newtonian orbital time $\tau=\epsilon t$ as a good time coordinate everywhere including inside the bodies. Without this nearly stationary internal solution, the natural time scale inside the bodies would be the free-fall time which scales like ϵ^2 and thus we would have to use the body's dynamical time $\eta=\epsilon^{-2}t$ inside the bodies. In such a case, a simple approximation scheme would not be possible.

In a previous paper[8] we used the above idea to introduce the point-particle limit[9] into the notion of a sequence of solutions and demonstrated the lowest-order calculation in the near and the far zones, giving Newton's equations of motion and the far-zone quadrupole formula. The quadrupole moment is expressed in terms of a mass integral over each compact star. The same mass appears in Newton's equations of motion. The mass is indeed the Arnowitt-Deser-Misner (ADM) mass[10] the compact star would have if it were isolated. In this paper we present a detailed calculation in the near zone up to radiation-reaction order and show that the rate of loss of Newtonian energy for orbital motion due to radiation reaction exactly balances the energy carried by the radiation in the far zone, as is given by the quadrupole formula. The energy loss is calculated both in a coordinate system in which the radiation-reaction force takes a very simple form as well as in the harmonic coordinate, thus explicitly showing the gauge invariance of the result. This paper provides the final step in our demonstration that the quadrupole formula may be used in the interpretation of the evolution of the binary pulsar system.

The organization of this paper is as follow: In Sec. II we shall formulate our scheme for a binary system and perform the lower-order calculation up to $O(\epsilon^5)$ and a part of $O(\epsilon^6)$. This calculation was already done in the previous paper,[8] but we present it here more clearly and for the sake of completeness of the present paper. In Sec. III we will give a detailed calculation of the radiation-reaction order. The resulting expression is used to calculate the energy loss in Sec. IV where the near-zone quadrupole formula is confirmed both in the harmonic gauge and the standard gauge. Also we shall derive the equation of spin precession. The strong equivalence principle for bodies with strong internal gravity is thus extended to the generation of gravitational radiation and the associated reaction effects and to the spin precession.

The notation in this paper will be the same as in our previous papers.[4] The basic equation is the reduced Einstein equation in harmonic coordinates:

$$\bar{h}^{\mu\nu}{}_{,\nu}=0 \ , \tag{1.1}$$

$$\Box\bar{h}^{\mu\nu}=-16\pi\Lambda^{\mu\nu} \ , \tag{1.2}$$

where $\bar{h}^{\mu\nu}=\eta^{\mu\nu}-(-g)^{1/2}g^{\mu\nu}$, η is the flat metric, $\Box=\eta^{\alpha\beta}\partial_\alpha\partial_\beta$,

$$\Lambda^{\mu\nu}=\Theta^{\mu\nu}+(16\pi)^{-1}(\bar{h}^{\alpha\nu}\bar{h}^{\beta\mu}-\bar{h}^{\alpha\beta}\bar{h}^{\mu\nu})_{,\alpha\beta} \ , \tag{1.3}$$

$$\Theta^{\mu\nu}=(-g)(T^{\mu\nu}+t_{\rm LL}{}^{\mu\nu}) \ . \tag{1.4}$$

$T^{\mu\nu}$ is the material stress-energy tensor and $t_{\rm LL}{}^{\mu\nu}$ is the Landau-Lifshitz pseudotensor. The near-zone field may be written formally as

$$\bar{h}^{\mu\nu}(\tau,x^k;\epsilon)=4\int_{c(\tau,x^k;\epsilon)}d^3y\,|x-y|^{-1}\times\underline{\Lambda}^{\mu\nu}(\tau-\epsilon|x-y|,y^k;\epsilon) \ , \tag{1.5}$$

where $c(\tau,x^k;\epsilon)$ is the past coordinate light cone of the event $(\tau/\epsilon,x^k)$ truncated at $t=0$ in the spacetime given by ϵ in the sequence. The quantity $\underline{\Lambda}^{\mu\nu}$ is related to $\Lambda^{\mu\nu}$ by $\underline{\Lambda}^{\mu\nu}(\tau,x^k;\epsilon)=\Lambda^{\mu\nu}(\tau/\epsilon,x^k;\epsilon)$. We have ignored the homogeneous solution by assuming zero data for the field which is motivated by the statistical argument advocated by Schutz.[11]

II. AN ASYMPTOTIC APPROXIMATION TO A BINARY PULSAR SYSTEM

A. Formulation

As discussed in the Introduction, we shall choose two uniformly rotating fluids with compact spatial support whose stress-energy tensor and size scale like ϵ^{-4} and ϵ^2, respectively, as our initial data to define the sequence of the solutions. This choice of the initial data leads naturally to the introduction of the body zones B_A and the body-zone coordinates $\alpha_A{}^{\underline{k}}$ defined by $B_A=\{x^k \| x^k-\xi^k_A\,|<\epsilon R\}$ for some R and $\alpha_A{}^{\underline{k}}=\epsilon^{-2}(x^k-\xi^k_A)$, respectively, where $\xi^k_A(\tau)$, $A=$I,II are the coordinates of the origin of the two bodies, where we have used letters with underlined indices to distinguish the body-zone coordinates from their near-zone counterparts. The scaling by ϵ^{-2} means that as the body shrinks with respect to x^i, it remains of fixed size in $\alpha^{\underline{k}}$. The boundary of the body zone shrinks to a point with respect to x^i, and expands to infinity with respect as $\alpha_A{}^{\underline{k}}$ as $\epsilon\rightarrow 0$. This makes a clean separation of the body from the exterior geometry generated by the other body (we have fixed the interbody separation).

Take two stationary solutions of Einstein's equations for the perfect fluid $\{T_A{}^{\mu\nu}(x^i),g^{\mu\nu}_A(x^i)\}$, $A=$I,II as our initial data in the body zone.[12] Every component of the stress-energy tensor $T_A{}^{\mu\nu}$ has the same ϵ^{-4} scaling in (t,x^k) coordinates for a rapidly rotating star. In the body-zone coordinates $(t,\alpha_A{}^{\underline{k}})$, these have the following scalings:

$$\left.\begin{aligned} T_A{}^{\tau\tau} &= \epsilon^2 T_A{}^{tt} \sim \epsilon^{-2} \\ T_A{}^{\tau\underline{i}} &= \epsilon^{-1} T_A{}^{ti} \sim \epsilon^{-5} \\ T_A{}^{\underline{ij}} &= \epsilon^{-4} T_A{}^{ij} \sim \epsilon^{-8} \end{aligned}\right\} \text{ for rapid rotation .} \tag{2.1a}$$

For a slowly rotating star, the strong gravity has to be supported by a large pressure gradient and the $T_A{}^{ij}$ scales the same was as $T_A{}^{tt}$ does. The only difference is the $T_A{}^{ti}$ which scales like ϵ^{-3}. Thus

$$T_A{}^{\tau\tau} \sim \epsilon^{-2}, \quad T_A{}^{\tau i} \sim \epsilon^{-4}, \quad T_A{}^{ij} \sim \epsilon^{-8} \quad \text{for slow rotation .} \tag{2.1b}$$

Transformed to the near-zone coordinates (τ, x^i), $x^i = \xi^i_A + \epsilon^2 \alpha_A{}^i$, they have the form

$$\begin{aligned} T_N{}^{\tau\tau} &= T_A{}^{\tau\tau} , \\ T_N{}^{\tau i} &= \epsilon^2 T_A{}^{\tau\underline{i}} + v_A{}^i T_A{}^{\tau\tau} , \\ T_N{}^{ij} &= \epsilon^4 T_A{}^{\underline{i}\underline{j}} + 2\epsilon^2 v_A{}^{(i} T_A{}^{j)\tau} + v_A{}^i v_A{}^j T_A{}^{\tau\tau} , \end{aligned} \tag{2.2}$$

where $v_A{}^i = d\xi^i_A / d\tau$ is the velocity of the origin. If there were only one body, these data would produce a stationary solution in the body zone, which moves with uniform velocity $v_A{}^i$ in the near-zone coordinates.

In this paper we will use the simpler order notation rather than derivatives with respect to ϵ. Since the boundaries of the body zones depend on ϵ, the actual calculation using derivative with respect to ϵ is rather tedious. On the other hand, the choice of the boundary is purely artificial and the result should not depend on the choice of the boundary. Any contribution from inside the body zones which depends on the choice of the boundaries should cancel with corresponding contribution from outside body zones. Therefore we do not need to take the ϵ dependence of the boundaries into account in the operation of derivatives with respect to ϵ or in the order notation. We use the order notation understanding that the more rigorous derivations may be substituted.

B. Near-zone calculation

According to the introduction of the body zones, the near-zone field (1.5) may be divided into two parts:

$$\bar{h}^{\mu\nu} = \bar{h}^{\mu\nu}{}_B + \bar{h}^{\mu\nu}{}_{C\setminus B} , \tag{2.3}$$

where $\bar{h}^{\mu\nu}{}_B$ is the contribution from integrating over the body zones, $\bar{h}^{\mu\nu}{}_{C\setminus B}$ from elsewhere. $\bar{h}^{\mu\nu}{}_B$ may be written as

$$\bar{h}^{\mu\nu}{}_B(\tau, x^k; \epsilon) = 4\epsilon^6 \sum_A \int_{B_A} d^3\alpha_A \, | x_A - \epsilon^2 \alpha_A |^{-1} \underline{\Lambda}^{\mu\nu}(\tau - \epsilon | x_A - \epsilon^2 \alpha_A |, \alpha_A{}^{\underline{k}}; \epsilon) , \tag{2.4}$$

where $x_A^i = x^i - \xi^i_A$ and we have used the body-zone coordinates for the integration variable. Expanding this in terms of ϵ, we have

$$\bar{h}^{\tau\tau}_B(\tau, x^k; \epsilon) = 4\epsilon^4 \sum_A \frac{M_A}{|x_A|} + O(\epsilon^6) , \tag{2.5}$$

$$\bar{h}^{\tau i}_B(\tau, x^k; \epsilon) = 4\epsilon^4 \sum_A \frac{J_A{}^i}{|x_A|} + 4\epsilon^4 \sum_A \frac{M_A v_A{}^i}{|x_A|} + O(\epsilon^5) , \tag{2.6}$$

$$\bar{h}^{ij}_B(\tau, x^k; \epsilon) = 4\epsilon^2 \sum_A \frac{Z_A{}^{ij}}{|x_A|} + 8\epsilon^4 \sum_A \frac{v_A{}^{(i} J_A{}^{j)}}{|x_A|} + 4\epsilon^4 \sum_A \frac{M_A v_A{}^i v_A{}^j}{|x_A|} + O(\epsilon^5) , \tag{2.7}$$

where

$$M_A \equiv \lim_{\epsilon \to 0} \epsilon^2 \int d^3\alpha_A \underline{\Lambda}^{\tau\tau}_A(\tau, \alpha_A{}^{\underline{k}}; \epsilon) , \tag{2.8}$$

$$J_A{}^i \equiv \lim_{\epsilon \to 0} \epsilon^4 \int d^3\alpha_A \underline{\Lambda}^{\tau\underline{i}}_A(\tau, \alpha_A{}^{\underline{k}}; \epsilon) , \tag{2.9}$$

$$Z_A{}^{ij} \equiv \lim_{\epsilon \to 0} \epsilon^4 \int d^3\alpha_A \underline{\Lambda}^{\underline{ij}}_A(\tau, \alpha_A{}^{\underline{k}}; \epsilon) , \tag{2.10}$$

where we have used the scaling (2.1b) for slow rotation since the binary pulsar is a slowly rotating neutron star. For systems composed of rapidly rotating stars, the definitions $J_A{}^i$ above and $J_A{}^{ij\cdots k}$ in (3.4) below are changed by the substitution of ϵ^3 instead of ϵ^4 in front of the integral in their definition. Thus terms involving $J_A{}^i$, $J_A{}^{ij\cdots k}$ in the metric derived below get another ϵ^{-1} factor for such a system. The M_A defined above is the (constant) ADM mass the body A would have if it were isolated. Without loss of generality one can set the linear momentum $J_A{}^i$ of the overall internal motion of each body to zero by choosing appropriate coordinates. If we did not assume the internal stationarity in the initial data, then $Z_A{}^{ij}$ would be finite and no approximation would be possible. However under the condition of internal stationarity one can show that $Z_A{}^{ij}$ vanishes. This can be seen from the relation

$$\int d^3\alpha_A \underline{\Lambda}^{\underline{ij}}_A = \frac{1}{2} \frac{d^2}{d\eta^2} \int d^3\alpha_A \alpha_A{}^{\underline{i}} \alpha_A{}^{\underline{j}} \Lambda^{\eta\eta} , \tag{2.11}$$

which is a consequence of $\underline{\Lambda}^{\mu\nu}{}_{,\nu} = 0$, where $\eta = \epsilon^{-2} t$ is the body's dynamical time. We have neglected the surface integrals which are of higher orders. Since the internal motion is governed by the Newtonian orbital time scale by our choice of the initial data, we have

$$\int d^3\alpha_A \underline{\Lambda}^{\underline{ij}}_A = \tfrac{1}{2} \epsilon^4 \frac{d^2}{d\tau^2} \int d^3\alpha_A \alpha_A{}^{\underline{i}} \alpha_A{}^{\underline{j}} \Lambda^{\tau\tau}_A . \tag{2.12}$$

This means that

$$Z_A^{ij}(\epsilon)\equiv\epsilon^4\int d^3\alpha_A\Lambda_A^{ij}(\tau,\alpha_A{}^k;\epsilon)$$
$$=\tfrac{1}{2}\epsilon^6 I_A^{(2)ij}(\epsilon)\ , \tag{2.13}$$

where we have defined the quadrupole moment of the body A as

$$I_A{}^{ij}(\epsilon)\equiv\epsilon^2\int d^3\alpha_A\alpha_A{}^i\alpha_A{}^j\Delta_A^{\tau\tau}\sim 1 \tag{2.14}$$

and $I^{(n)}\equiv(d^n/d\tau^n)I$. Therefore the first term in (2.7) is actually of order ϵ^8 and the first nonvanishing effect of the variation of the quadrupole moment of the component stars will be due to tidal effects and will appear at order ϵ^8, which is of third post-Newtonian order. The above expression for $\bar{h}_B{}^{\mu\nu}$ is used to calculate the pseudotensor

$$(-g)t_{\rm LL}{}^{\mu i}=O(\epsilon^6)\ ,$$
$$(-g)t_{\rm LL}{}^{ij}=\epsilon^4(64\pi)^{-1}({}_4\bar{h}^{\tau\tau,i}{}_4\bar{h}^{\tau\tau,j}-\tfrac{1}{2}\delta^{ij}{}_4\bar{h}^{\tau\tau,k}{}_4\bar{h}^{\tau\tau}{}_{,k})$$
$$+O(\epsilon^6)\ . \tag{2.15}$$

This pseudotensor is used to calculate the contribution $\bar{h}_{C\setminus B}{}^{\mu\nu}$ from outside the body zones.

The resulting metric deviation up to $O(\epsilon^8)$ is given by

$$\bar{h}^{\tau\tau}=4\epsilon^4\sum_A\frac{M_A}{|x_A|}+O(\epsilon^6)\ , \tag{2.16a}$$

$$\bar{h}^{\tau i}=4\epsilon^4\sum_A\frac{M_Av_A{}^i}{|x_A|}+2\epsilon^6\sum_A\frac{x_A^k}{|x_A|}M_A{}^{ki}+O(\epsilon^6)\ , \tag{2.16b}$$

$$\bar{h}^{ij}=4\epsilon^4\left[\sum_A\frac{M_Av_A{}^iv_A{}^j}{|x_A|}+\lim_{\epsilon\to0}\epsilon^{-4}\int_{C\setminus B}d^3y\,|x-y|^{-1}t_{\rm LL}{}^{ij}(\tau,y;\epsilon)\right]-2\epsilon^5I_{\rm orb}^{(3)ij}+4\epsilon^6\sum_A\frac{x_A^k}{|x_A|^3}v_A{}^{(i}M_A{}^{|k|j)}+O(\epsilon^6)\ , \tag{2.16c}$$

where

$$M_A{}^{ij}\equiv\lim_{\epsilon\to0}\epsilon^4 2\int_{B_A}d^3\alpha_A\alpha_A^{[i}\Delta_A^{j]\tau}\ , \tag{2.17}$$

$$I_{\rm orb}^{ij}\equiv\sum_A\xi^i{}_A\xi^j{}_AM_A\ . \tag{2.18}$$

These are the angular momentum of the body A and the quadrupole moment for the orbital motion, respectively. We have included a part of $O(\epsilon^6)$ terms which depend on the angular momentum since these terms are used later to calculate the equation of spin precession. In the above derivation we have used the relation

$$J_A^{ij}(\epsilon)\equiv\epsilon^3\int_{B_A}d^3\alpha_A\alpha_A{}^i\Delta_A{}^{\tau j}$$
$$=\tfrac{1}{2}[M_A^{ij}(\epsilon)+\epsilon^3I_A^{(1)ij}(\epsilon)]\ , \tag{2.19}$$

where $M_A^{ij}(\epsilon)$ is the quantity inside the limiting sign in (2.17).

In order to see how the quadrupole moment for the orbital motion appears in (2.16c), we shall show an explicit derivation of the order-ϵ^5 term in $\bar{h}^{ij}$. First the ϵ^5-order term in $\bar{h}_B{}^{ij}$ is given by

$${}_5\bar{h}_B{}^{ij}=-4\sum_A\frac{d}{d\tau}(M_Av_A{}^iv_A{}^j)\ . \tag{2.20}$$

This is combined with the order-ϵ^5 contribution from $\bar{h}^{ij}{}_{C\setminus B}$:

$${}_5\bar{h}^{ij}{}_{C\setminus B}=-4\int_{C\setminus B}d^3y\frac{\partial}{\partial\tau}{}_4[(-g)t_{\rm LL}{}^{ij}]\ . \tag{2.21}$$

Using the expression (2.15) for $(-g)t_{\rm LL}{}^{ij}$, we have

$$\int_{C\setminus B}d^3y\,{}_4(-g)t_{\rm LL}{}^{ij}=-\oint_{\partial B}d^2S_ky^j{}_4[(-g)t_{\rm LL}{}^{ik}]+\int_{C\setminus B}d^3y\,y^j{}_4[(-g)t_{\rm LL}{}^{ik}]_{,k}$$
$$=-\sum_A\oint_{\partial B_A}d^2S_k(\xi_A^j+r_An^j)\,{}_4t_{\rm LL}{}^{ik}$$
$$=-2\frac{M_{\rm I}M_{\rm II}}{R}N^iN^j$$
$$=2\sum_A M_Av_A^{(1)(i}\xi_A{}^{j)}\ , \tag{2.22}$$

where $N^i=R^i/R$, $R^i=\xi^i_{\rm I}-\xi^i_{\rm II}$, and we have used the fact that ${}_4[(-g)t^{ik}]_{,k}=0$ on ∂B_A and neglected the surface integral at infinity which is irrelevant. We have also used the Newton's equations of motion derived in the previous paper using the surface integral over the boundary of the body zones. Combining (2.20) and (2.22), we arrive at the expression in (2.16c).

III. RADIATION REACTION ORDER

We shall now calculate higher-order terms which are relevant in the radiation-reaction calculation. These are terms with the odd order in ϵ up to $O(\epsilon^9)$ in $\bar{h}^{\tau\tau}$ and $O(\epsilon^7)$ in $\bar{h}^{\tau i},\bar{h}^{ij}$. In the calculation we shall make use of relations such as (2.13) and (2.19) which express the condition of internal stationarity. These are

$$J_A{}^{ijk}(\epsilon)=\tfrac{2}{3}M_A{}^{(ij)k}(\epsilon)+\tfrac{1}{3}\epsilon^3 I_A^{(1)ijk} \; , \tag{3.1}$$

$$Z_A{}^{ijk}(\epsilon)=\tfrac{1}{2}\epsilon^6 I_A{}^{(2)ij}(\epsilon)-\epsilon^3 P_A^{(1)ijk}(\epsilon) \; , \tag{3.2}$$

and so on, where

$$I_A{}^{ij\cdots k}(\epsilon)\equiv\epsilon^2\int_{B_A} d^3\alpha_A\,\alpha_A{}^i\alpha_A{}^j\cdots\alpha_A{}^k\underline{\Delta}_A{}^{\tau\tau} \; , \tag{3.3}$$

$$J_A{}^{ij\cdots k}(\epsilon)\equiv\epsilon^4\int_{B_A} d^3\alpha_A\,\alpha_A{}^i\alpha_A{}^j\cdots\underline{\Delta}_A{}^{\tau k} \; , \tag{3.4}$$

$$Z_A{}^{ij\cdots kl}(\epsilon)\equiv\epsilon^4\int_{B_A} d^3\alpha_A\,\alpha_A{}^i\alpha_A{}^j\cdots\underline{\Delta}_A{}^{kl} \; , \tag{3.5}$$

are higher moments of the components stars. Using these relations, we may express any moments of the star in terms of the mass moments and current moments. The higher current moments are defined as

$$M_A{}^{ij\cdots kl}(\epsilon)=\epsilon^4 2\int_{B_A} d^3\alpha_A\,\alpha_A{}^i\alpha_A{}^j\cdots\alpha_A{}^{[k}\underline{\Delta}_A{}^{l]\tau} \; . \tag{3.6}$$

Using the above relation at each stage of the calculation, one finds that the contributions from the body zone at order ϵ^7 are given as

$$_7\bar{h}^{\tau\tau}{}_B=-\frac{4}{3}\sum_A\frac{d}{d\tau}(M_A v_A{}^2) \; , \tag{3.7}$$

$$_7\bar{h}^{\tau i}{}_B=+\frac{4}{3}\sum_A x_A{}^k\frac{d^3}{d\tau^3}(M_A\xi_A{}^k v_A{}^i) \; , \tag{3.8}$$

$$_7\bar{h}^{ij}{}_B=-\frac{2}{3}\sum_A x_A{}^2\frac{d^3}{d\tau^3}(M_A v_A{}^i v_A{}^j) \; . \tag{3.9}$$

These are combined with the contributions from outside the body zones:

$$_7\bar{h}^{\tau\tau}{}_{C\setminus B}=\lim_{\epsilon\to 0}\left[-\tfrac{4}{3}\int_{C\setminus B} d^3y\;{}_4[(-g)t_{\rm LL}^{(1)kk}]\right] , \tag{3.10}$$

$$\begin{aligned}_7\bar{h}^{\tau i}{}_{C\setminus B}=\lim_{\epsilon\to 0}\Big[&\tfrac{4}{3}x^k\int_{C\setminus B} d^3y\;{}_4[(-g)t_{\rm LL}^{(2)ik}]\\ &-\tfrac{4}{3}\int_{C\setminus B} d^3y\,y^k\;{}_4[(-g)t_{\rm LL}^{(2)ik}]\Big] ,\end{aligned} \tag{3.11}$$

$$\begin{aligned}_7\bar{h}^{ij}{}_{C\setminus B}=\lim_{\epsilon\to 0}\Big[&-\tfrac{2}{3}x^2\int_{C\setminus B} d^3y\;{}_4[(-g)t_{\rm LL}^{(3)ij}]\\ &+\tfrac{4}{3}x^k\int_{C\setminus B} d^3y\,y^k\;{}_4[(-g)t_{\rm LL}^{(3)ij}]\\ &-\tfrac{2}{3}\int_{C\setminus B} d^3y\,y^2\;{}_4[(-g)t_{\rm LL}^{(3)ij}]\\ &-4\int_{C\setminus B} d^3y\;{}_6[(-g)t_{\rm LL}^{(1)ij}]\Big] .\end{aligned} \tag{3.12}$$

Terms in (3.11) and (3.12) which are only functions of time do not contribute to the Landau-Lifshitz pseudotensor at the radiation-reaction order. The contribution in the energy loss from the second term in (3.12) also vanishes in the center-of-mass frame. Therefore the relevant expressions in the calculation of energy loss are given by

$$_7\bar{h}^{\tau\tau}=-\tfrac{2}{3}I_{\rm orb}^{(3)kk} \; , \tag{3.13}$$

$$_7\bar{h}^{\tau i}=+\tfrac{2}{3}x^k I_{\rm orb}^{(4)ik} \; , \tag{3.14}$$

$$_7\bar{h}^{ij}=-\tfrac{1}{3}x^2 I_{\rm orb}^{(5)ij} \; , \tag{3.15}$$

where we have used (2.20) in (3.10)–(3.12).

The order ϵ^9 in $\bar{h}^{\tau\tau}$ is similarly calculated and is given by

$$\begin{aligned}_9\bar{h}^{\tau\tau}= &-\tfrac{4}{15}\sum_A x_A^k x_A^l\frac{d^3}{d\tau^3}[M_A(v_A{}^k v_A{}^l+\tfrac{1}{2}\delta^{kl}v_A{}^2)]\\ &+\lim_{\epsilon\to 0}\Big[-\tfrac{4}{15}x^i x^j\int_{C\setminus B} d^3y\{{}_4[(-g)t_{\rm LL}^{(3)ij}]+\tfrac{1}{2}\delta^{ij}\,{}_4[(-g)t_{\rm LL}^{(3)kk}]\}\\ &+\tfrac{8}{15}x^i\int_{C\setminus B} d^3y\{2y^k\,{}_4[(-g)t_{\rm LL}^{(3)ik}]+y^i\,{}_4[(-g)t_{\rm LL}^{(3)kk}]\}+\tfrac{4}{15}\int_{C\setminus B} d^3y(y^iy^j+\tfrac{1}{2}\delta^{ij}y^2)\,{}_4[(-g)t_{\rm LL}^{(3)ij}]\\ &-\tfrac{4}{3}\int_{C\setminus B} d^3y\;{}_6[(-g)t_{\rm LL}^{(1)\tau\tau}]+4\int_{C\setminus B} d^2y\,|x-y|^{-1}\,{}_9\underline{\Delta}^{\tau\tau}\Big] .\end{aligned} \tag{3.16}$$

For the same reasons explained in the calculation of $_7\bar{h}^{\mu\nu}$ the third, fourth, and fifth terms do not contribute in the energy-loss calculation. The first and second terms may be combined into the following form using (2.20):

$$-\tfrac{2}{15}x^k x^l(I_{\rm orb}^{(5)kl}+\tfrac{1}{2}\delta^{kl}I_{\rm orb}^{(5)aa}) \tag{3.17}$$

plus other terms which do not contribute in the energy loss for the same reason above. The last term in (3.16) does have a nonvanishing contribution. Since $_9\underline{\Delta}^{\tau\tau}$ takes the form

$$_9\underline{\Delta}^{\tau\tau}=(16\pi)^{-1}({}_5\bar{h}^{kl}\,{}_4\bar{h}^{\tau\tau})_{,kl} \; , \tag{3.18}$$

where $_5\bar{h}^{kl}=-2I_{\rm orb}^{(3)kl}$, the last term satisfies the equation

$$\Delta\phi={}_5\bar{h}^{kl}\,{}_4\bar{h}^{\tau\tau}{}_{,kl} \; . \tag{3.19}$$

The solution is easily found to be

$$\phi={}_5\bar{h}^{kl}\chi_{,kl} \; , \tag{3.20}$$

where $\chi=\sum_A M_A|x_A|$. In the end we have

$$_9\bar{h}^{\tau\tau}=-\tfrac{2}{15}x^k x^l(I_{\rm orb}^{(5)kl}+\tfrac{1}{2}\delta^{kl}I_{\rm orb}^{(5)aa})-2I_{\rm orb}^{(3)kl}\chi_{,kl} \; . \tag{3.21}$$

The expressions (3.13), (3.18), (3.15), and (3.21) are used in the next section to calculate the radiation-reaction effect.

IV. THE QUADRUPOLE FORMULA AND THE SPIN PRECESSION

A. The quadrupole formula

We first calculate the energy loss due to radiation reaction. We define the four-momentum of body A as

$$P_A{}^{\mu}=\epsilon^2 \int_{B_A} d^3\alpha_A \underline{\Theta}^{\mu\tau}(\tau,x^k) \ . \tag{4.1}$$

Then its time derivative is given by the surface integral

$$\frac{d}{d\tau}P_A{}^{\mu}=-\epsilon^2 \oint_{\partial B_A} d^2S_k \underline{\Theta}^{\mu k}+\epsilon^2 v_A{}^k \oint_{\partial B_A} d^2S_k \underline{\Theta}^{\mu\tau} \ , \tag{4.2}$$

where $\Theta^{\mu\nu}=(-g)t_{\mathrm{LL}}{}^{\mu\nu}$ on ∂B_A, the boundary of the body zone B_A. The second term arises because the body zone moves with velocity $v_A{}^i=d\xi^i_A/d\tau$. Any ambiguities in the definition of $P_A{}^{\mu}$ due to the location of ∂B_A or the choice of ξ^k_A go away as $\epsilon \rightarrow 0$ at the order we shall consider.

For the space component of (4.1), $P_A{}^i$ may be divided into two parts according to (2.2): $\underline{\Theta}^{i\tau}=\epsilon^2\underline{\Theta}^{i\tau}+v_A{}^i\underline{\Theta}^{\tau\tau}_A$. The first part coming from $\underline{\Theta}^{i\tau}_A$ which is the momentum associated with the internal motion of the body A vanishes for our choice of coordinate system (see Sec. II B). Therefore $P_A{}^{\mu}$ is expressed as

$$P_A{}^i=v_A{}^iP_A{}^{\tau} \ . \tag{4.3}$$

The surface integral of (4.2) is easily calculated once one has the explicit expression for $(-g)t_{\mathrm{LL}}{}^{\mu\nu}$ up to the radiation-reaction order. We already know that $P_A{}^{\tau}$ is conserved up to second post-Newtonian order[13] and we have

$${}_9[(-g)t_{\mathrm{LL}}{}^{\tau\tau}]={}_9[(-g)t_{\mathrm{LL}}{}^{\tau i}]=0 \ . \tag{4.4}$$

Therefore

$$\frac{d}{d\tau}P_A{}^{\tau}=0 \tag{4.5}$$

up to the radiation-reaction order. In fact this is the ADM mass (2.8) the body A would have if it were isolated:

$$P_A{}^{\tau}=M_A+O(\epsilon^{10}) \ . \tag{4.6}$$

The calculation of ${}_9[(-g)t_{\mathrm{LL}}{}^{ij}]$ is rather tricky. Not only quadratic terms but also cubic terms contribute at ϵ^9 order.[14] The relevant terms are

$$\begin{aligned}16\pi\, {}_9[(-g)t_{\mathrm{LL}}{}^{ij}]={}&\tfrac{1}{4}[\,2\,{}_4\bar{h}^{\tau\tau,(i}({}_9\bar{h}^{|\tau\tau|,j)}+{}_7\bar{h}^{k,j)}_{k})-\delta^{ij}\,{}_4\bar{h}^{\tau\tau,k}({}_9\bar{h}^{\tau\tau}_{,k}+{}_7\bar{h}^{l}_{l,k})]\\ &+2\,{}_4\bar{h}^{\tau\tau,(i}\,{}_7\bar{h}^{j)\tau}{}_{,\tau}-\delta^{ij}\,{}_4\bar{h}^{\tau\tau,k}\,{}_7\bar{h}^{\tau}{}_{k,\tau}-\tfrac{3}{4}\delta^{ij}\,{}_4\bar{h}^{\tau\tau}{}_{,\tau}\,{}_7\bar{h}^{\tau\tau}{}_{,\tau}\\ &-2\,{}_4\bar{h}^{\tau k,(i}\,{}_5\bar{h}^{j)}{}_{k,\tau}+\delta^{ij}\,{}_4\bar{h}^{\tau k,l}\,{}_5\bar{h}_{lk,\tau}+\tfrac{1}{4}\delta^{ij}\,{}_4\bar{h}^{\tau\tau}{}_{,\tau}\,{}_5\bar{h}^{k}{}_{k,\tau}\\ &+\tfrac{1}{8}(4\delta^{k(i}\,{}_5g^{j)l}-{}_5g^{ij}\delta^{kl}-\delta^{ij}\,{}_5g^{kl})\,{}_4\bar{h}^{\tau\tau}{}_{,k}\,{}_4\bar{h}^{\tau\tau}{}_{,l}-\tfrac{1}{4}(2\delta^{ik}\delta^{jl}-\delta^{ij}\delta^{kl})\,{}_3g_{\tau\tau}\,{}_4\bar{h}^{\tau\tau}{}_{,k}\,{}_4\bar{h}^{\tau\tau}{}_{,l} \ ,\end{aligned} \tag{4.7}$$

where ${}_5g^{ij}=-2\not{I}^{(3)\,ij}_{\mathrm{orb}}\ {}_3g_{\tau\tau}=-\tfrac{4}{3}I^{(3)\,kk}_{\mathrm{orb}}$. Straightforward calculation of the surface integral in (4.2) gives the equations of motion with radiation reaction in the harmonic coordinates:

$$M_A\frac{dv_A{}^i}{d\tau}=-\frac{M_{\mathrm{I}}M_{\mathrm{II}}}{R^3}R^i+\epsilon^5\left[\tfrac{3}{5}M_A\xi^k_A\not{I}^{(5)ik}+2M_Av_A{}^kI^{(4)ik}-\frac{1}{3}\frac{M_{\mathrm{I}}M_{\mathrm{II}}}{R^3}I^{(3)a}_{a}R^i-3\frac{M_{\mathrm{I}}M_{\mathrm{II}}}{R^5}R^kR^lR^iI^{(3)kl}\right] . \tag{4.8}$$

We have neglected the first and second post-Newtonian terms since these do not contribute the lowest-order energy loss due to radiation emission.[15]

We now calculate the rate of change of the total Newtonian energy, $E_N=\frac{1}{2}\sum_A M_Av_A{}^2-M_{\mathrm{I}}M_{\mathrm{II}}/R$, due to radiation reaction in the harmonic coordinate:

$$\begin{aligned}\frac{d}{d\tau}E_N&=\sum_A\left[\frac{dv_A{}^i}{d\tau}\frac{\partial E_N}{\partial v_A}+\frac{d\xi^i_A}{d\tau}\frac{\partial E_N}{\partial \xi^i_A}\right]\\ &=\epsilon^5\left[\tfrac{3}{10}\not{I}^{(1)\,ij}_{\mathrm{orb}}\not{I}^{(5)\,ij}_{\mathrm{orb}}-\tfrac{1}{2}\not{I}^{(3)\,ij}_{\mathrm{orb}}\not{I}_{\mathrm{orb}}{}^{(3)ij}+2\sum_A M_Av_A{}^kv_A{}^iI^{(4)\,ik}_{\mathrm{orb}}+4\frac{M_{\mathrm{I}}M_{\mathrm{II}}}{R^3}R^{(i}V^{j)}I^{(3)\,ij}_{\mathrm{orb}}\right] ,\end{aligned} \tag{4.9}$$

where $V^i=v_{\mathrm{I}}{}^i-v_{\mathrm{II}}{}^i$ and we have used the equation of motion (4.8) and the following expression for $I^{(3)\,ij}_{\mathrm{orb}}$:

$$I^{(3)ij}_{\mathrm{orb}}=-8\frac{M_{\mathrm{I}}M_{\mathrm{II}}}{R^3}V^{(i}R^{j)}+6\frac{M_{\mathrm{I}}M_{\mathrm{II}}}{R^5}(V\cdot R)R^iR^j \ . \tag{4.10}$$

By averaging in time, we have

$$\left\langle\frac{d}{dt}E_N\right\rangle=-\tfrac{1}{5}\epsilon^4\langle\not{I}^{(3)\,ij}_{\mathrm{orb}}\not{I}^{(3)\,ij}_{\mathrm{orb}}\rangle \ . \tag{4.11}$$

This exactly balances the energy carried away by the gravitational radiation in the far zone.[8] The observable effect, which is the period shortening in the case of the binary pulsar system, may be calculated from (4.11). This is because only the lowest-order period functional is required to calculate the (gauge-invariant) lowest-order period change, so we need only the Newtonian period P_N, which is a function of the Newtonian energy E_N (Ref. 15). Thus the near-zone quadrupole formula (4.11) immediately predicts the period decay.

The rate of change of the total angular momentum due to radiation reaction is also calculated in terms of the surface integral

$$\frac{d}{d\tau}S_{\mathrm{total}}{}^{a}=-\epsilon^{a}{}_{bi}\int_{\partial N}d^{2}S_{j}x^{b}(-g)t_{\mathrm{LL}}{}^{ij}\ , \tag{4.12}$$

where the total angular momentum is defined as

$$S_{\mathrm{total}}{}^{a}=\epsilon^{a}{}_{bi}\int_{N}d^{3}x\,x^{b}\underline{\Theta}^{\tau i}\ . \tag{4.13}$$

The integral is over the entire near zone and thus the surface integral (4.12) is done over the boundary of the near zone. The relevant expression for $\bar{h}^{\mu\nu}$ for a large distance from the source is

$$\bar{h}^{\tau\tau}=6\epsilon^{4}\frac{x^{k}x^{l}}{r^{5}}\not{I}^{kl}_{\mathrm{orb}}-\tfrac{2}{3}\epsilon^{7}I^{(3)}_{\mathrm{orb}}{}^{kk}$$
$$-\tfrac{2}{15}\epsilon^{9}x^{k}x^{l}(I^{(5)}_{\mathrm{orb}}{}^{kl}+\tfrac{1}{2}\delta^{kl}I^{(5)}_{\mathrm{orb}}{}^{aa})\ , \tag{4.14}$$

$$\bar{h}^{\tau i}=2\epsilon^{5}\frac{x^{k}}{r^{3}}I^{(1)}_{\mathrm{orb}}{}^{ki}+\tfrac{2}{3}\epsilon^{7}x^{k}I^{(4)}_{\mathrm{orb}}{}^{ki}\ , \tag{4.15}$$

$$\bar{h}^{ij}=-2\epsilon^{(5)}I^{(3)}_{\mathrm{orb}}{}^{ij}-\tfrac{1}{3}\epsilon^{7}x^{2}I^{(5)}_{\mathrm{orb}}{}^{ij}\ . \tag{4.16}$$

Straightforward calculation gives

$$\frac{d}{d\tau}S_{\mathrm{total}}{}^{a}=-\epsilon^{5}\epsilon^{a}{}_{bi}(-\tfrac{9}{25}\not{I}^{(5)}_{\mathrm{orb}}{}^{ki}\not{I}^{kb}_{\mathrm{orb}}+\tfrac{6}{25}\not{I}^{(5)}_{\mathrm{orb}}{}^{kb}\not{I}^{ki}_{\mathrm{orb}}$$
$$+\tfrac{1}{3}\not{I}^{(4)}_{\mathrm{orb}}{}^{bk}\not{I}^{(1)}_{\mathrm{orb}}{}^{ki}-\tfrac{2}{3}\not{I}^{(1)}_{\mathrm{orb}}{}^{bk}\not{I}^{(4)}_{\mathrm{orb}}{}^{ki})\ . \tag{4.17}$$

After time averaging we get the near-zone formula

$$\left\langle\frac{dS_{\mathrm{total}}{}^{a}}{d\tau}\right\rangle=-\tfrac{2}{5}\epsilon^{5}\epsilon^{a}{}_{bi}\langle\not{I}^{(2)}_{\mathrm{orb}}{}^{kb}\not{I}^{(3)}_{\mathrm{orb}}{}^{ki}\rangle \tag{4.18}$$

which is consistent with the amount of the angular momentum transported by the radiation in the far zone.[16]

B. The standard coordinate

The radiation-reaction force (4.8) derived in the harmonic coordinate depends on the velocity and acceleration of the particle as well as the position, and it takes a form that is remarkably similar to that in the linearized theory.[11] Therefore it is impossible to treat the reaction force as an additional Newtonian-type force in the harmonic coordinate. Although the reaction force is coordinate dependent and thus has no particular physical meaning, it is nice to have a simple expression which allows one to have a simple interpretation.

We shall find the coordinate system in which the reaction force takes the familiar expression. By the following coordinate transformation,

$$x^{\mu'}=x^{\mu}+f^{\mu}\ , \tag{4.19}$$

the metric deviation $\bar{h}^{\mu\nu}$ transforms as

$$\bar{h}^{\mu\nu'}=\bar{h}^{\mu\nu}-(\eta^{\mu\rho}-\bar{h}^{\mu\rho})f^{\nu}{}_{,\rho}-(\eta^{\nu\rho}-\bar{h}^{\nu\rho})f^{\mu}{}_{,\rho}$$
$$+(\eta^{\mu\nu}-\bar{h}^{\mu\nu})f^{\rho}{}_{,\rho}-f^{\rho}\bar{h}^{\mu\nu}{}_{,\rho}\ . \tag{4.20}$$

The coordinate of the origin of the body A transforms as well:

$$x^{k'}(\tau')=\xi^{k}_{A}(\tau)+f^{k}{}_{,l}\xi^{l}{}_{A}(\tau)\ ; \tag{4.21}$$

i.e., $\xi^{i}{}_{A}(\tau)$ transforms as a three-vector in the spatial transformation. By choosing the gauge function f^{μ},

$$f^{\tau}=\epsilon^{5}\tfrac{2}{3}I^{(2)}_{\mathrm{orb}}{}^{kk}+\epsilon^{7}{}_{7}f^{\tau}\ , \tag{4.22}$$

$$f^{i}=-\epsilon^{5}x^{k}I^{(3)}_{\mathrm{orb}}{}^{ki}+\epsilon^{7}{}_{7}f^{i}\ , \tag{4.23}$$

where ${}_{7}f^{\mu}$ are arbitrary functions (the effects of ${}_{7}f^{\mu}$ exactly cancel in the result), one finds

$$\begin{aligned}
&{}_{5}g^{ij'}={}_{3}g'_{\tau\tau}={}_{5}\bar{h}^{ij'}={}_{7}\bar{h}^{\tau\tau'}=0\ ,\\
&{}_{7}\bar{h}^{\tau i'}={}_{7}\bar{h}^{\tau i}-x^{k}I^{(4)}_{\mathrm{orb}}{}^{ki}\ ,\\
&{}_{7}\bar{h}^{ij'}={}_{7}\bar{h}^{ij}\ ,\\
&{}_{9}\bar{h}^{\tau\tau'}=-\tfrac{2}{15}x^{k}x^{l}(I^{(5)}_{\mathrm{orb}}{}^{kl}+\tfrac{1}{2}\delta^{kl}I^{(5)}_{\mathrm{orb}}{}^{aa})-4I^{(3)}_{\mathrm{orb}}{}^{kl}\chi_{,kl}\\
&\qquad+\tfrac{2}{3}I^{(3)}_{\mathrm{orb}}{}^{kk}\,{}_{4}\bar{h}^{\tau\tau}-\tfrac{2}{3}I^{(2)}_{\mathrm{orb}}{}^{kk}\,{}_{4}\bar{h}^{\tau\tau}{}_{,\tau}+x^{l}I^{(3)}_{\mathrm{orb}}{}^{kl}\,{}_{4}\bar{h}^{\tau\tau}{}_{,k}\ .
\end{aligned} \tag{4.24}$$

Using these expressions in the calculation of the surface integral (4.2) and taking the change of the position of the particle (4.14) into account in the Newtonian force term, we finally get the required result

$$M_{A}\frac{d^{2}}{d\tau'^{2}}X_{A}{}^{i'}(\tau')=-\frac{M_{\mathrm{I}}M_{\mathrm{II}}}{R'^{3}}R^{i'}$$
$$-\tfrac{2}{5}\epsilon^{5}M_{A}X_{A}{}^{k'}(\tau')\not{I}^{(5)}_{\mathrm{orb}}{}^{ik} \tag{4.25}$$

where $R^{i'}=X_{\mathrm{I}}{}^{i'}-X_{\mathrm{II}}{}^{i'}(\tau')$.

Therefore, just as in the linearized[11] and usual (weak gravity) post-Newtonian cases,[17] it is possible to treat the problem as an effectively nonrelativistic one even for systems with strong (nearly stationary) internal gravity, in which one pretends that the radiation-reaction force is just another Newtonian-type potential force. But this is possible only in the very special coordinate system described above.

C. The spin precession

Since our scheme can take consistently the rotation of the component star into account, it is natural to ask how the equation for spin precession is derived in our scheme. Just as the equation of motion is derived in terms of surface integrals, the spin precession is also derived in terms of surface integrals. We define the spin four-vector of the body A as

$$S_{A\rho}=\tfrac{1}{2}\epsilon_{\mu\nu\rho\sigma}M_{A}{}^{\mu\nu}u^{\sigma}_{A}\ , \tag{4.26}$$

where u^{σ}_{A} is the four-velocity of the body A and $M_{A}{}^{\mu\nu}$ is defined as

$$M_{A}{}^{\mu\nu}(\epsilon)=\epsilon^{4}2\int_{B_{A}}d^{3}\alpha_{A}\alpha_{A}{}^{[\mu}\underline{\Theta}^{\nu]\tau}\ . \tag{4.27}$$

The spatial component of the spin vector is thus given by

$$S_{Aa}=-\tilde{S}_{Aa}u^{\tau}+\epsilon_{ija}M_{A}{}^{i\tau}u_{A}{}^{j}\ , \tag{4.28}$$

where we have defined the spatial spin vector $\tilde{S}_{Aa}$ as

$$\tilde{S}_{Aa}=\tfrac{1}{2}\epsilon_{ija}M_A{}^{ij}(\epsilon) \ . \tag{4.29}$$

The time derivative of the spatial part of the spin four-vector is therefore given by

$$\frac{d}{d\tau}S_{Aa}=-\frac{d}{d\tau}\tilde{S}_{Aa}\cdot u_A{}^{\tau}-\tilde{S}_{Aa}\frac{d}{d\tau}u_A{}^{\tau}+\epsilon_{ija}\left[\frac{d}{d\tau}M_A{}^{i\tau}\cdot u_A{}^{j}+M_A{}^{i\tau}\frac{d}{d\tau}u_A{}^{j}\right] \tag{4.30}$$

$$=-\epsilon\frac{d}{d\tau}\tilde{S}_{Aa}-\epsilon^3\tilde{S}_{Aa}\frac{d}{d\tau}(\tfrac{1}{2}v_A{}^2+\phi)+\epsilon\epsilon_{ija}v_A{}^{j}\frac{d}{d\tau}M_A{}^{i\tau}+\epsilon\epsilon_{ija}M_A{}^{i\tau}\frac{d}{d\tau}v_A{}^{j} \ , \tag{4.31}$$

where we have neglected irrelevant terms and used $u_A{}^{\mu}=\epsilon(dx_A{}^{\mu}/d\tau)[1+\epsilon^2(\frac{1}{2}v_A{}^2+\phi)+O(\epsilon^4)]$ in (4.31); $\phi=-M_B/R$ is the potential generated by the other body B. The time derivative of $\tilde{S}_{Aa}$ is calculated by the surface integral as[18]

$$\frac{d}{d\tau}\tilde{S}_{Aa}=-\epsilon_{abc}\oint_{\partial B_A}d^2S_d\alpha_A{}^{b}\underline{\Theta}^{cd}+\epsilon_{abc}v_A{}^{d}\oint_{\partial B_A}d^2S_d\alpha_A{}^{b}\underline{\Theta}^{cc} \ . \tag{4.32}$$

Following the standard deviation,[19] we shall make the time transformation

$$t_{\text{new}}=t_{\text{old}}+\epsilon^4\frac{1}{2}\sum_A\frac{M_Ar_A{}^{k}v_A{}^{k}}{r_A} \ . \tag{4.33}$$

The only relevant change in $\bar{h}^{\alpha\beta}$ in the calculation is $\bar{h}^{\tau i}$ at order ϵ^4:

$${}_4\bar{h}^{\tau i}_{\text{new}}=\frac{7}{2}\sum_A\frac{M_Av_A{}^{i}}{r_A}+\frac{1}{2}\sum_A\frac{M_Av_A{}^{k}r_A{}^{k}r_A{}^{i}}{r_A{}^3} \ . \tag{4.34}$$

Using (2.16) and (4.34),

$$\frac{d}{d\tau}\tilde{S}_{Aa}=\epsilon^2\epsilon_{abi}(\phi^{,b}v_A{}^{k}M_A{}^{ki}-\tfrac{4}{3}\phi^{,k}v_A{}^{k}M_B{}^{ki})+\epsilon^2\epsilon_{abi}(-\tfrac{4}{3}v_B{}^{[i}\phi^{,k]}M_A{}^{kb}+\tfrac{8}{3}v_B{}^{[b}\phi^{,k]}M_B{}^{k})$$
$$+\epsilon^3\epsilon_{abi}\frac{3M_A{}^{ki}}{R^3}[(N^kN^l-\tfrac{1}{3}\delta^{kl})M_B{}^{lb}-(N^bN^l-\tfrac{1}{3}\delta^{lb})M_B{}^{lk}] \ . \tag{4.35}$$

The time derivative of $M_A{}^{i\tau}$ is similarly calculated by the surface integral and we get

$$\frac{d}{d\tau}M_A{}^{i\tau}=-\epsilon^2\tfrac{2}{3}\phi^{,k}M_A{}^{ki} \ . \tag{4.36}$$

Notice that $M_A{}^{i\tau}=D_A{}^{i}-\tau J_A{}^{i}=0$ in our choice of the coordinate system, where $D_A{}^{i}$ is the dipole moment of the body A, $J_A{}^{i}$ is the linear momentum of the overall internal motion defined in (2.9).

The final result may be put in the form

$$\frac{d\mathbf{S}_A}{d\tau}=(\mathbf{\Omega}_A{}^{\text{geod}}+\mathbf{\Omega}_A{}^{\text{LT}})\times\mathbf{S}_A \ , \tag{4.37}$$

where

$$\mathbf{\Omega}_A{}^{\text{geod}}=\epsilon^3\frac{M_B}{R^2}(2\mathbf{v}_B-\tfrac{3}{2}\mathbf{v}_A)\times\mathbf{N} \ , \tag{4.38}$$

$$\mathbf{\Omega}_A{}^{\text{LT}}=\epsilon^4\frac{1}{R^3}[-\mathbf{S}_B+3\mathbf{N}(\mathbf{N}\cdot\mathbf{S}_B)] \ . \tag{4.39}$$

These are the angular velocity of geodetic precession and Lense-Thirring precession, respectively. In (4.37) $\mathbf{S}_A$ has components on the orthonormal spatial basis vectors of the local asymptotic rest frame of body A rather than $S_A{}^{a}$: namely,

$$S_A{}^{\hat{a}}=\left[1+\frac{M_{\text{II}}}{R^2}\right]S_A{}^{a}-\tfrac{1}{2}v_A{}^{a}v_A{}^{b}S_A{}^{b} \ . \tag{4.40}$$

Therefore we get the same result derived in the test particle case using the Papapetrou equation[20] and that derived in the weak gravity case.[21] The same result for possible strong gravity was also obtained by Thorne and Hartle[22] using the multipole moment method.

V. DISCUSSION

We have constructed a sequence of solutions which is the basis of an asymptotic approximation in general relativity to a binary system composed of rotating neutron stars. Unfortunately, a mathematically rigorous proof of the asymptotic nature of the approximation does not exist since no rigorous error estimate is available at the moment. It seems that such a proof for realistic sources may not be available in the near future.

The strong internal gravity is accommodated in our scheme by introducing a point-particle limit. It has been said that the point particle, strictly speaking, does not exist in general relativity, but by means of the limiting procedure along a sequence of solutions we can consistently use the notion of a rotating point particle within the framework of general relativity, in the limiting sense of a body whose dimensions are small compared to the relevant dynamical (orbital) scale, yet which possesses strong internal gravity.

In our scheme we have assumed nonsingular initial data (two neutron stars, not black holes) that are defined by the volume integral over the body. In this way we are able to extend the strong equivalence principle up to radiation-reaction order. We cannot, however, apply our scheme to black holes. In this respect, the Einstein-Infeld-Hoffmann (EIH) scheme[23] has been generalized by Anderson up to radiation-reaction order[24] using a procedure which contains only surface integrals at any stage. The method is very similar to our scheme, and the result coincides.

In future work, it may be interesting to calculate the octopole radiation-reaction terms because of the recent suggestion that it may be possible to determine the Hubble constant by observing gravitational waves from a coalescing binary system.[25] The ratio of octopole radiation flux to the quadrupole radiation flux for such a system may become more than 10% and might be observable. Far-zone calculations at octopole radiation order has been performed by Wagoner and Will using δ-function-type sources.[26]

It would also be interesting to apply this point-particle limit in the fast-motion case.[27] When applied to electromagnetism, it provides the Lorentz-Dirac equation for a relativistic charged particle with *finite*-mass renormalization.[28]

ACKNOWLEDGMENTS

I would like to thank David Garfinkle, Bernard Schutz, and Clifford Will for valuable discussions and suggestions. I wish to thank James Anderson for pointing out a mistake in the original version of this paper and for discussions on his EIH method. This work was commenced while I was in the Department of Applied Mathematics and Astronomy, University College, Cardiff, for whose kind hospitality I am grateful. I wish to thank the McDonnell Center for the Space Sciences and Department of Physics, Washington University, St. Louis, for kind hospitality. This work was supported by the National Science Foundation under Grants Nos. PHY 83-13545 and PHY 85-13953.

---

[1]J. H Taylor, L. A. Fowler, and P. M. Mulloch, Nature (London) **377**, 939 (1979).

[2]J. Ehlers, L. Rosenblum, J. N. Goldberg, and P. Havas, Astrophys. J. **208**, 477 (1976); J. Ehlers, Ann. N.Y. Acad. Sci. **336**, 279 (1980); M. Walker and C. M. Will, Phys. Rev. Lett. **45**, 1741 (1980).

[3]J. L. Anderson, Phys. Rev. Lett. **45**, 1745 (1980); R. A. Breuer and E. Rudolph, Gen. Relativ. Gravit. **13**, 777 (1981); T. Damour, in *Gravitational Radiation,* edited by N. Deruelle and T. Piran (North-Holland, Amsterdam, 1983). See also T. Damour, Phys. Rev. Lett. **51**, 1019 (1983); R. A. Isaacson, J. S. Welling, and J. Winicour, *ibid.* **53**, 1870 (1984); R. Kates, Phys. Rev. D **22**, 1871 (1980); G. P. Kerlick, Gen. Relativ. Gravit. **12**, 467 (1980); **12**, 521 (1980); A. Papapetrou and B. Linet, *ibid.* **13**, 335 (1981); S. Persides, *ibid.* (to be published); G. Schäfer, Prog. Theor. Phys. **38**, 2191 (1982); J. Winicour, Max-Planck Institut für Astrophysik report, 1986 (unpublished).

[4]T. Futamase and B. F. Schutz, Phys. Rev. D **28**, 2363 (1983); T. Futamase, *ibid.* **28**, 2373 (1983); T. Futamase and B. F. Schutz, *ibid.* **32**, 2557 (1985).

[5]There is a criticism of this statement from a mathematically rigorous point of view by J. Ehlers, in *General Relativity and Gravitation,* edited by B. Bertotti, F. deFelice, and A. Pascolini (Reidel, Holland, 1984).

[6]See, for example, C. M. Will, *Theory and Experiment in Gravitational Physics* (Cambridge University Press, Cambridge, England, 1981).

[7]There is other evidence in special cases, in post-post-Newtonian order and the black-hole case; G. Börner and E. Rudolph, Gen. Relativ. Gravit. **9**, 809 (1978); P. D'Eath, Phys. Rev. D **12**, 2183 (1975).

[8]T. Futamase, Phys. Rev. D **32**, 2566 (1985).

[9]P. D'Eath, Phys. Rev. D **11**, 1387 (1975); R. Kates, *ibid.* **22**, 1871 (1980).

[10]R. Arnowitt, S. Deser, and C. W. Misner, in *Gravitation: An Introduction to Current Research,* edited by L. Witten (Wiley, New York, 1962).

[11]B. F. Schutz, Phys. Rev. D **22**, 249 (1980).

[12]I wish to thank Bernard Schutz for suggesting to employ this way of formulating initial data.

[13]S. Chandrasekhar, Astrophys. J. **142**, 1488 (1965).

[14]I wish to thank James Anderson for pointing out the necessity of the cubic terms.

[15]Futamase (Ref. 4); B. F. Schutz, *Relativistic Astrophysics and Cosmology,* edited by X. Fustero and E. Verdaguer (World Scientific, Singapore, 1984).

[16]B. S. DeWitt (unpublished).

[17]M. Walker and C. M. Will, Astrophys. J. **242**, L129 (1980).

[18]I wish to thank Clifford Will for pointing out the importance of the second term in (4.32).

[19]See, for example, C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973).

[20]A. Papapetrou, Proc. R. Soc. London **A209**, 248 (1951).

[21]B. M. Barker and R. F. O'Connell, Phys. Rev. D **2**, 1428 (1970).

[22]K. S. Thorne and J. Hartle, Phys. Rev. D **31**, 1815 (1985).

[23]A. Einstein, L. Infeld, and B. Hoffmann, Ann. Math. **39**, 65 (1938).

[24]J. Anderson, in Proceedings of the 11th International Conference on General Relativity and Gravitation, 1986 (unpublished), p. 468.

[25]B. F. Schutz, Nature (London) **323**, 310 (1986).

[26]R. V. Wagoner and C. M. Will, Astrophys. J. **210**, 764 (1976).

[27]A. Rosenblum, Phys. Rev. Lett. **81**, 1003 (1978).

[28]T. Futamase (in preparation).